* 1. -Graph

A digraph can be a useful device for representing relations and functions, especially if the relation isn’t “too large” or complicated.

Definition

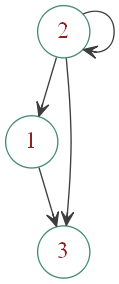
* A directed graph or a digraph **[1]** *D* from *A* to *B* is a collection of vertices *V* ⊆ *A* ∪ *B* and a collection of edges *R* ⊆ *A* × *B*.
* If there is an ordered pair *e* = (*x*, *y*) in *R* then there is an arc or edge from *x* to *y* in *D*.
* The elements *x* and *y* are called the initial and terminal vertices of the edge *e* = (*x*, *y*), respectively.
* The nodes in the digraph represent pairs of a relation and edges between these nodes represent the relation between nodes.

A diagraph can also be illustrated visually as demonstrated by the following examples of previous section

Digraph of example 2.3.1:

*A* = {1, 2, 3}

*R* = {(1, 3), (2, 1), (2, 2), (2, 3)}

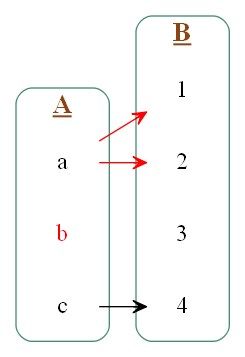


**Figure. 2.1 di-graph for relation R**

Digraph of example 2.3.2:

*A* = {a, b, c} and *B* = {1, 2, 3, 4}

*R* = {(a, 1), (a, 2), (c, 4)}



**Figure. 2.2 di-graph for relation *R***

* 1. Properties of Homogeneous Binary Relations

The homogeneous binary relations have certain properties like reflexive, symmetric, antisymmetric and transitive. The detail about these properties are given below:

Let *A* be a set, and let *R* be a binary relation on *A*

* + 1. *R is* **reflexive [1]** *if*

*∀x*[(*x ∈ A*) *→* ((*x, x*) *∈ R*)]

* + 1. *R is* **symmetric [1]** *if*  
       *∀x∀y*[((*x, y*) *∈ R*) *→* ((*y, x*) *∈ R*)]
    2. *R is* **antisymmetric [1]** *if*  
       *∀x∀y*[([(*x, y*) *∈ R*] *∧* [(*y, x*) *∈ R*]) *→* (*x* = *y*)]

or, equivalently,

*∀x∀y*[([(*x, y*) *∈ R*] *∧* (*a ≠ b*)) *→ (*(*x, y*) ∉ *R)*]

* + 1. *R is* **transitive [1]** *if*  
       *∀x∀y∀z*[([(*x, y*) *∈ R*] *∧* [(*y, z*) *∈ R*]) *→* ((*x, z*) *∈ R*)]

In informal terms

1. **Reflexive:** Each element is related to itself.
2. **Symmetric:** If any one element is related to any other element, then the second element is related to the first.
3. **Transitive:** If any one element is related to a second and that second element is related to a third, then the first element is related to the third.

Sometime The transitivity condition is vacuously true for *R.* To see this, observe that the transitivity condition says that

For all *x, y, z* ∈ *A,* if *(x, y)* ∈ *T* and *(y, z)* ∈ *T* then *(x, z)* ∈ *T.*

The only way for this to be false would be for there to exist elements of *A* that make the hypothesis true and the conclusion false. That is, there would have to be elements *x, y,* and *z* in *A* such that

*(x, y)* ∈ *T* and *(y, z)* ∈ *T* and *(x, z) ∉* *T.*

In other words, there would have to be two ordered pairs in *R* that have the potential to “link up” by having the *second* element of one pair be the *first* element of the other pair. It follows that it is impossible for *R not* to be transitive, and thus *R* is transitive.

**Chapter 3**

**3. Functions**

* 1. Introduction to Functions

A function is something that associates each element of a set with an element of another set (which may or may not be the same as the first set). For example, a social security number uniquely identifies the person, the income tax rate varies depending on the income, the final letter grade for a course is often determined by test and exam scores, home works and projects, and so on.

In [mathematics](https://en.wikipedia.org/wiki/Mathematics), a function is a [relation](https://en.wikipedia.org/wiki/Binary_relation) between a [set](https://en.wikipedia.org/wiki/Set_(mathematics)) of inputs and a set of permissible outputs with the property that each input is related to exactly one output. An example is the function that relates each real number *x* to its square *x*2. The output of a function *f* corresponding to an input *x* is denoted by *f*(*x*) (read "*f* of *x*"). In this example, if the input is −3, then the output is 9, and we may write *f*(−3) = 9. Likewise, if the input is 3, then the output is also 9, and we may write *f*(3) = 9.

* 1. Definition of a Function

A function **[2]** *F* from a set *A* to a set *B*is a relation with domain *A* and co-domain *B* that satisfies the following two properties:

1. For every element *x* in *A*, there is an element *y* in *B* such that *(x, y)* ∈ *F*.
2. For all elements *x* in *A* and *y* and *z* in *B*, if *(x, y)* ∈ *F* and *(x, z)* ∈ *F,* then *y*=*z.*

Properties (1) and (2) can be stated less formally as follows: A relation *F* from *A* to *B* is a function if, and only if:

1. Every element of *A* is the first element of an ordered pair of *F*.
2. No two distinct ordered pairs in *F* have the same first element.

The set of all values of *f* taken together is called the range of *f* or the image of *A* under *f*.